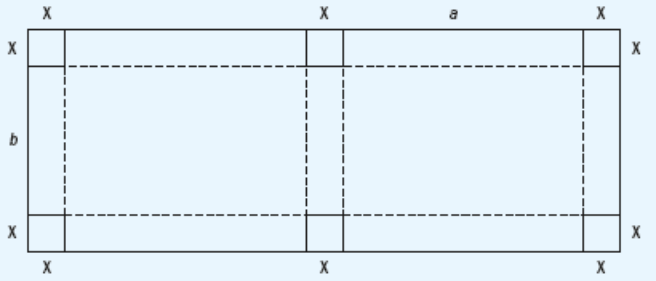
Chapter 4 / **Example 34**

Optimization problems

A piece of cardboard, measuring 100 cm by 200 cm, is to be made into a box by cutting out small squares, each with side length x, as shown in the diagram. a is the length between the squares on the longer side of the cardboard, and b is the length between the squares on the shorter side of the cardboard.

i Find expressions for a and b in terms of x, and state the constraints on the lengths of x, a, and b.

ii Find the value of x (in cm^3) which maximizes the volume of the box, and find the maximum volume of the box. Check your answers graphically.



The problem requires the maximization of the function

$$V = x(100 - 2x)\left(\frac{200 - 3x}{2}\right) \quad x < 50$$

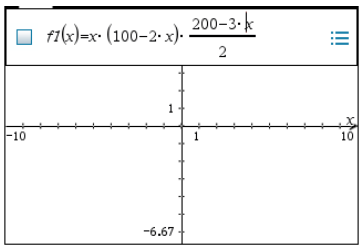
Open a new document and add a Graphs page.

The entry line is displayed at the top of the work area.

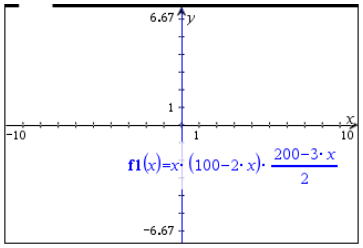
The default graph type is function, so '$f1(x)=$' is displayed.

The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.

Type $x \times (100 - 2x) \times \left(\frac{200 - 3x}{2}\right)$ and press enter.



The GDC displays the graph $f1(x) = x(100 - 2x)\left(\frac{200 - 3x}{2}\right)$ with the default axes.



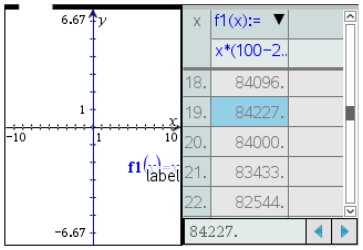
Chapter 4 / **Example 34**

Optimization problems

To get a better idea of the best window to view the graph in, it is helpful to use a table of values. Press ctrl T.

A table of values is displayed alongside the graph.

You can scroll through the table using ▲ and ▼ on the touchpad.



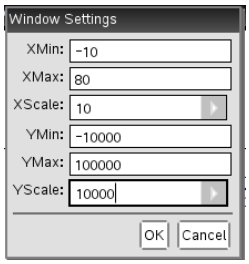
Use this information to choose suitable window settings to display the graph.

Press ctrl T again to remove the table.

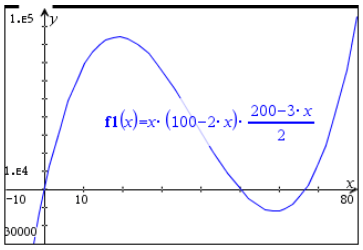
Press menu 4:Window/Zoom | 1:Window Settings…

Set the axes to show $-10 \le x \le 80$ with a scale of 10 and $-30\,000 \le y \le 100\,000$ with a scale of 10 000

Press enter when you have finished.



The GDC displays the curve in a suitable window.

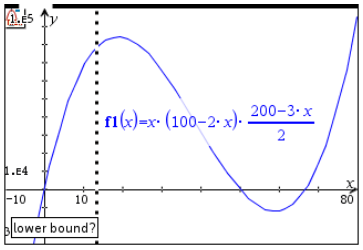


To find the maximum press menu 6:Analyse Graph | 3:Maximum.

You will need to give the lower and upper bounds of the region that includes the maximum.

The GDC shows a line and asks you to set the lower bound. Move the line using the touchpad and choose a position to the left of the maximum.

Click the touchpad.

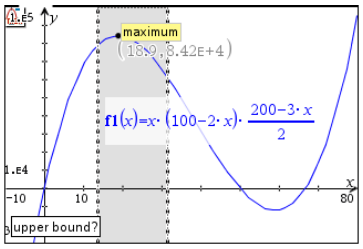


The GDC shows another line and asks you to set the upper bound.

Use the touchpad to move the line so that the region between the lower and upper bounds contains the maximum.

When the region contains the maximum, the calculator will display the word 'maximum' in a box.

Click the touchpad.



Chapter 4 / **Example 34**

Optimization problems

The GDC displays the local maximum point at $(18.9, 84\,200)$.

There is also a minimum point which occurs when $x > 50$ which can be rejected.

So the maximum volume is 84 200 cm^3 when x = 18.9 cm.